An Exotic Approach to Hadron Physics

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An exotic approach to hadrons is discussed. It is based on the recently developed open-closed string duality explicitly conjectured as the AdS/CFT correspondence. Mesons as well as pentaquarks are studied in this approach. Spins are introduced as distribution functions over the string, and a second quantization method of string theory is examined and used to estimate the mass and decay width of various hadrons. This approach provides a way to understand the structure of flavor by a configuration of probe branes.

§1. Introduction

The standard way to study hadrons is to estimate a hadron mass as the sum of quark masses and spin-spin or hyperfine interactions. To study the excitation states one uses a phenomenological linear plus Coulomb potential.

An exotic approach for this is to consider string theory in a curved space which is the s-channel and t-channel dual to QCD,[1)] where spins are distributed over the string.

Strings come in both open and closed form. Open strings have two end points, each of which can be labelled with a Chan-Paton factor giving a flavor or color quantum number. If both ends carry color, the string transforms in the adjoint of the colour group, like the gluon. If one end carries color and the other end flavor, the string transforms in the fundamental of the color group and fundamental of the flavour group, like a quark. If both ends carry flavor, the string transforms as a meson. The classical energy of a string is proportional to its length, so that, classically, the length of massless gluons is vanishing, but those of heavier quarks and mesons, corresponding to stretched strings, are not. The splitting and merging of zero length strings at the color end gives the emission and absorption of gluons, namely, the gauge interactions of color (QCD). Similarly the splitting and merging of non-zero length strings at the flavor end gives the gauge interaction of flavor, where vector mesons, such as ρ and K*, are gauge fields, but the gauge symmetry is broken by their non-zero masses.

The graviton is one of the massless modes of the closed string. The reason is as

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follows: an open string can make a small circle at any point along the string, this circle can be split from the string, propagates and is absorbed to another string. The probability of emitting or absorbing of this circle is proportional to the length (area) of the emitter or the absorber. Therefore, the closed string coupling to an open string or an extended object is proportional to the mass (length, area or volume) of the object. The closed string has a number of modes, such as the graviton and the antisymmetric field, but their couplings (or charge) to an extended object are thus proportional to the mass of the object.

By placing a stack of N_c color branes and $N_f (\ll N_c)$ flavor branes in ten-dimensional space the respective Chan-Paton factors of the strings attached to these branes give the correct transformation properties for gluons, quarks and mesons. When the branes are separated the stretched strings acquire a mass. By keeping the color branes parallel the gluons remain massless but massive quarks and hadrons can be introduced into this picture by the separation of the flavor branes.

If the brane is the object of tieing the endpoint of open string. The brane can be a textile made of the strings as threads. Then, the tension T_p of a p-brane (the energy per unit volume of a p-dimensionally extended object) is proportional to $(T_1)^{(p+1)/2}$ by dimensional counting, where T_1 is the tension of the original string. The existence of massive color and flavor branes distorts the extra space gravitationally. For $N_f << N_c$ we can treat this distortion as coming only from the color branes and treat the flavor branes as probes. It is found that the appropriate solution to the supergravity equations gives a space which retains the Minkowski directions parallel to the D3 color branes and warps the space in the directions perpendicular to these branes.

Let us draw a number of circles on the color branes and flavor branes, and draw a surface with these circles as its boundary. This is a configuration of string theory contributing to a process of QCD. See for example Fig. 3 of Ref. 3). The s-channel
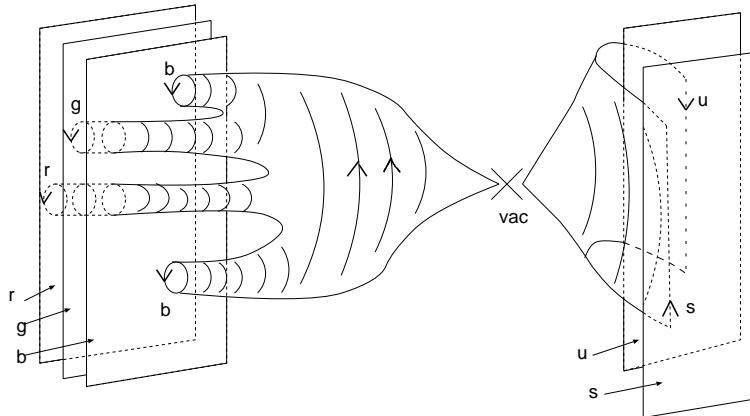

Fig. 1. Estimation of force between u and $\bar{s}$.

view is to see the picture in the time direction inside the brane. The t-channel view is to see this in the direction of the extra space. Here, we can understand that "the number of loops in the s-channel@view" (in the open string theory) is replaced by "the number of external lines in the t-channel view" (in the closed

string theory). Therefore, "sum over loops in the open string theory" is replaced by "sum over external lines at tree level of closed string theory", so that the non-perturbative QCD is replaced by the classical gravity, where the sum over external lines terminating on the color branes gives the classical contribution of gravity from infinite points on the branes. Of course the open string theory and the closed string theory are different from QCD and a certain gravitational theory, respectively. In order to obtain classical supergravity decoupled from the flat-space region, we look at the near horizon geometry with the string tension taken to infinity, N_c taken to infinity and the 't Hooft coupling $N_c g_s$ taken to be a large constant. This classical theory of supergravity corresponds to a strongly coupled large N_c field theory.[1] To obtain QCD we need to keep $N_c = 3$ and therefore to consider the $1/N_c$ corrections properly. Another important issue is to break supersymmety by either turning on appropriate supergravity fields or by embedding the branes in a configuration which eliminates the supercharges.

A candidate for the t-channel view of realistic QCD is described in a deformed space of "6 dimensional AdS Schwarzshild space" x S^4.[2] In this model, color branes are spatially extended along the four directions $(\mathbf{x}_\perp, z, \vartheta)$, while flavor branes are spatially extended along the six directions $(\mathbf{x}_\perp, z, x^5, x^6, x^7)$. The ϑ direction is compactified on a circle with radius $1/M_{KK}$ (a free parameter). The antiperiodic boundary conditions of the fermions around the circle give them a mass and break supersymmetry. The deformation of the space comes only from the stuck N_c heavy color branes, but not from flavor branes. This is a reasonable assumption, since we want to include the non-perturbative (multi-loop) effects of QCD, but not of QFD. (If we consider the gravitational@effects from the flavor branes, the non-perturbative effects of QFD would be included.)

The metric of this deformed space is given by using $u = \sqrt{(x^5)^2 + \cdots (x^9)^2}$,

$$ds^2 = f(u)(-dt^2 + dz^2 + d\mathbf{x}_\perp^2) + g(u)du^2 + g(u)^{-1}d\vartheta^2 + f(u)^{-1}u^2 d\Omega_4^2, \quad (1\cdot1)$$

where

$$f(u) = (u/R')^{3/2}, \quad g(u) = (f(u)h(u))^{-1}, \quad h(u) = 1 - (U_{KK}/u)^3, \quad (1\cdot2)$$

and $d\Omega_4^2$ is the metric on the S^4 with unit radius. R' and U_{KK} are given by using QCD coupling α_c as

$$R'^3 = 2\pi\alpha_c N_c/M_{KK}, \quad \text{and} \quad U_{KK} = \frac{8\pi}{9}\alpha_c N_c M_{KK}. \quad (1\cdot3)$$

§2. Pentaquark Θ^+

In this model, pentaquark baryons are studied.[3] The problem is to find catenary lines in the above deformed space, by pinning down five end points of lines (strings) to the five flavors ($udud\bar{s}$) on the respective flavor branes. Here, the strings should be connected at three junctions ($N_c = 3$). The mass formulae was obtained roughly. A bonus in this study is obtaining an indication for the small decay width: In order for a pentaquark to decay into a baryon and a meson, it should pass through a

heavier state with baryon or meson with a string loop. See the following Fig. 6 of Ref. 3). String theoretical estimation of the decay width following this figure is an
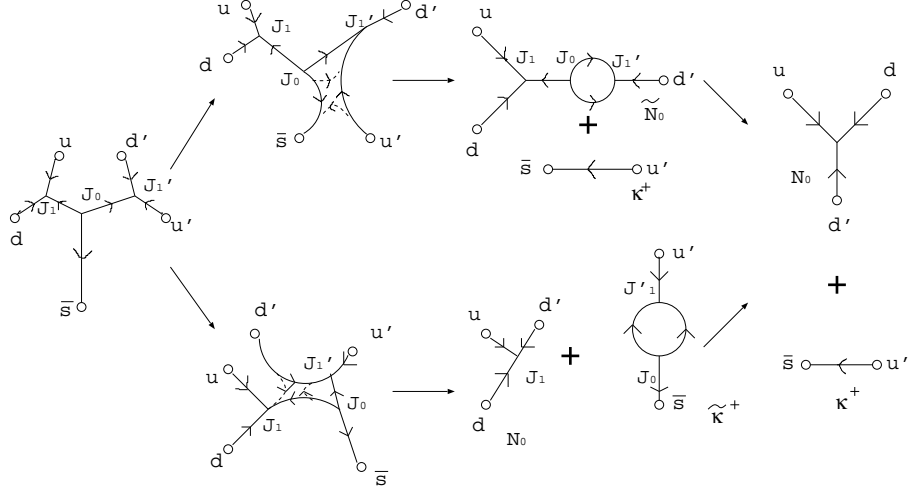

Fig. 2.   Decay process of the pentaquark Θ^+ in string theory.

open problem.

§3.   Meson Strings and Flavor Branes

Next, meson strings are studied in detail.[4] There is a clear difference between mesons with the same (mass) flavors ($u\bar{u}, u\bar{d}, s\bar{s}, ...$) and mesons with different flavors ($u\bar{s}, u\bar{b}, d\bar{t}, ...$). The following figures show that for mesons with different flavors, there is a critical distance where the shape of the string connecting the branes changes. This critical point is also the point of inflection of the quark and anti-quark potential.
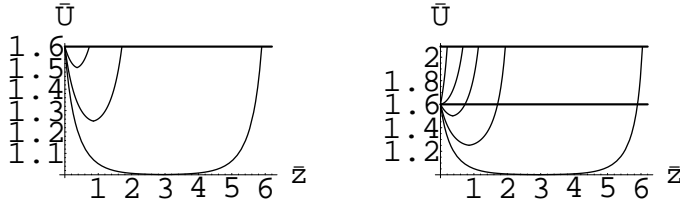

Fig. 3.   Profile of $u\bar{d}$ (left) and $u\bar{s}$ (right) strings.

Namely, the potentials are different between mesons with light flavors and those with heavy and light flavors. Physical implications of this phenomenon are to be studied in detail.
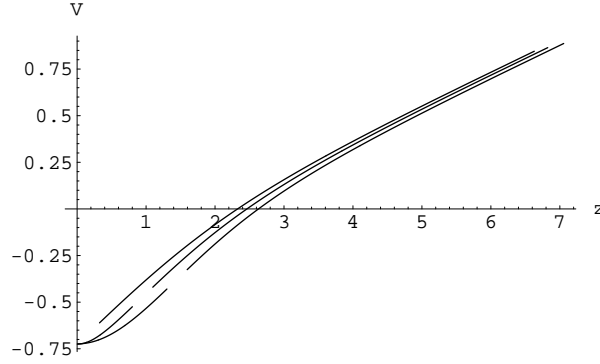
Fig. 4. Potential as a function of $r = Z \text{ GeV}^{-1}$ for the mesons $u\bar{d}$(the upper curve), $u\bar{s}$ (the middle curve) and $u\bar{b}$ (the lower curve).

§4. Introduction of Spins into Hadrons

We know from deep inelastic scattering experiment that quarks carry only half of nucleon momentum and only 38 percent of nucleon spin. The former is a common sense and the latter is a "spin crisis".[5] Using string theory we may be able to find a solution to this crisis, since both momentum and spin can be distributed over strings (or hadron) by two variables, $X^\mu(\tau, \sigma)$ and $\psi^\mu(\tau, \sigma)$. The ψ satisfies a commutation relation,

$$\{\psi^\mu(\tau, \sigma), \psi^\nu(\tau, \sigma')\} = \eta^{\mu\nu}\delta(\sigma - \sigma'), \tag{4.1}$$

so that $\psi^\mu(\tau, \sigma)/\sqrt{2}$ can be a distribution function of γ^μ matrix. Therefore, spin is distributed over the string theory. The action in our deformed space may be given, generalizing the work by Iwasaki and Kikkawa,[6] as

$$S = \int d^2\xi \sqrt{-det g_{ab}} \;, \tag{4.2}$$

with a world sheet metric

$$g_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(x) + \frac{i}{2}\left(\bar{\psi}^\mu \hat{e}_a D_b \psi^\nu - D_b \bar{\psi}^\mu \hat{e}_a \psi^\nu\right) G_{\mu\nu}(x) \;, \tag{4.3}$$

where $\hat{e}_a = e_{a\bar{a}}\gamma^{\bar{a}}$ is the contraction of a zwiebein by a two dimensional γ matrix, and

$$D_a \psi^\mu = \partial_a \psi^\mu + \partial_a X^\nu \Gamma^\mu_{\nu\lambda}(x)\psi^\lambda \;. \tag{4.4}$$

The action includes quadratic as well as quartic terms for ψ^μ, leading to the spin-spin or hyperfine interactions. A preliminary study[7] shows that the energy of the meson changes considerably by the excitation of spin degrees of freedom.

§5. Utilization of Second Quantization of Strings[8]

To quantize a particle, we start from a constraint for its energy and momentum,

$$E - \frac{1}{2m}p^2 - V(x) = 0 \;, \tag{5.1}$$

replace E by $i\frac{d}{dt}$ and p by $-i\frac{d}{dx}$ to satisfy the commutation relations and apply the obtained constraint operator to a wave function $\psi(t,x)$ of the particle. Then we arrive at the quantum theory

$$\left[i\frac{d}{dt} - \frac{1}{2m}\left(i\frac{d}{dt} \right)^2 - V(x) \right]\psi(t,x) = 0 . \tag{5·2}$$

In the same manner, (second) quantization of a string can be performed with the constraint equations,

$$P^M P_M + \left(\frac{1}{2\pi\alpha'} \right)^2 (X')^2 = 0 , \quad X'^M P_M = 0 . \tag{5·3}$$

The string is an extended object, so its momentum is a functional differential, $P_M(\tau,\sigma) = \frac{-i\delta}{\delta X^M(\tau,\sigma)}$. Now the wave equations of the string $\Psi(X^N(\tau,\sigma))$ can be obtained as follows:

$$\left[G^{MN}(x)\left(\frac{-i\delta}{\delta X^M(\tau,\sigma)} \right)\left(\frac{-i\delta}{\delta X^N(\tau,\sigma)} \right) + \left(\frac{1}{2\pi\alpha'} \right)^2 G_{MN}(x)(X'^M)(X'^N) \right]$$
$$\times \Psi(X^N(\tau,\sigma)) = 0 , \tag{5·4}$$

$$X'_M(\tau,\sigma)\left(\frac{-i\delta}{\delta X^N(\tau,\sigma)} \right)\Psi(X^M(\tau,\sigma)) = 0 . \tag{5·5}$$

Here the background metric $G_{MN}(x)$ is to be the deformed one corresponding to the supergravity dual of large N_c QCD. We know the classical static configuration of hadrons, such as mesons as well as pentaquarks, in the deformed space as catenary lines. As we know, the wave function of the WKB approximation for a particle takes the following form

$$\psi = e^{-i(Et + W_0 + iW_1 + W_2 + iW_3 + \cdots)}, \tag{5·6}$$

where W_0 is a classical action at rest, and the exponents of wave function, being expanded in terms of $\hbar$, are alternately real and imaginary. Therefore, in the string theory we set

$$\Psi(\tau, X^I(\sigma)) = e^{-i(E\tau - P_z z - \mathbf{P}_\perp \cdot \mathbf{x}_\perp + E_{cl}(z)\tau)}\tilde{\Psi}(\tau, X^I(\sigma)) , \tag{5·7}$$

with

$$\tilde{\Psi}(\tau, X^I(\sigma)) = e^{A - iB} . \tag{5·8}$$

Here, $E_{cl}(z)$ is the static energy of a given string configuration (depending on the perpendicular length z of each string segment), so that $E_{cl}(z)\tau$ is the classical action of the string at rest, which corresponds to W_0 for a particle. Therefore, the wave equations determine the real and imaginary exponents A and B of wave function as quantum corrections. So, we hope to determine the mass of hadrons as well@as the decay width of them in this formulation. The existence of a critical point found for a certain configurations of heavy and light mesons suggests an unstable configuration at a saddle point, from which we may estimate the decay rate of the meson as a tunneling probability. We will clarify these points in the near future.

§6. Deformation of Flavor Branes (Running Behavior of Quark Mass)

Under the gravitational force from the stuck N_c color branes, the flavor branes may change their shape. This is a very important issue. The extra dimension $\lambda = \sqrt{(x^5)^2 + \cdots + (x^7)^2}$ are internal coordinates of the flavor branes, while $r = \sqrt{(x^8)^2 + (x^9)^2}$ are perpendicular to the flavor brane worldvolume. Before supersymmetries are broken, there is no force between color branes and flavor branes and $r(\lambda)$ is constant dynamically, but after the symmetries are broken flavor branes are deformed in the UV as[2]

$$r(\lambda) \xrightarrow[\lambda \to \infty]{} m_{\text{current}} + \frac{\langle \bar{\psi}\psi \rangle}{\lambda} . \tag{6.1}$$

Here, r is the distance between color branes and the flavor brane, so that it is related to the quark mass with this flavor. The value of the quark bilinear condensate is zero in the supersymmetric case but can be non-zero in the non-supersymmetric deformations. The change of r according to the change of λ from ∞ to zero is understood as a running behavior of the quark mass from UV to IR. By finding
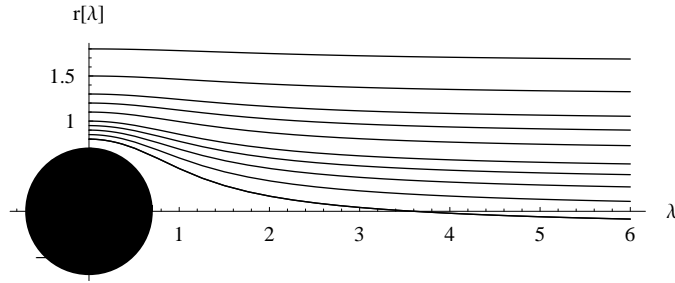

Fig. 5. Deformation of a D6-brane probe in the type IIA supergravity background created by a stack of N_c D4-branes with a compactified spatial dimension.

the well-behaved solutions for a given limiting value in the UV, the quark bilinear condensate and quark mass can be read from the UV asymptotics $r = m + \frac{c}{\lambda}$. The lowest line in the figure is a metastable flow which can be continuously deformed into a negative condensate solution. The renormalisation group flow of the quark mass is still somewhat ill-defined in this holographic context though in Ref. 9) an attempt was made to define a gauge invariant measure of the RG energy scale. The circular disk gives the region which is not truly a part of the space due to the compactification.

§7. Energy of String Junction

Another important issue is the energy of string junctions. In our study of pentaquarks we have considered no energy for them. The energy of junction in the QCD like model is estimated by Imamura[10] as

$$E_J = c\frac{\pi N_c}{(2\pi)^4}u, \quad c = -0.874 . \tag{7.1}$$

This is -12 MeV, for $M_{KK} = 0.77$ GeV which reproduces the Regge slope of hadrons. Therefore, junctions may stabilize the exotic fullerene or ferrule like compounds made of strings and junctions.[11]

§8. Summary

(1) An open string with color and flavor at its ends is a good picture of a quark.

(2) Flavor structure may be understood in the picture of locating flavor branes in the extra space.

(3) Then, the shape of the potential between heavy and light quarks gives a point of inflection.

(4) Spin may be distributed over the whole string.

(5) Treatment of kinetic terms in the estimation of hadron masses is improved by the second quantization method of the string. Then, the mass as well as the decay width of hadrons may be estimated.

(6) The radial direction perpendicular to the D3-branes is dual to an energy scale in the field theory and the holographic renormalisation group flow of the quark mass must be understood.

(7) There is evidence that the junction has negative energy.

(8) It is interesting to study the recently observed various exotic hadrons, or the further exotics such as fullerene and ferrule with our exotic approach given here. This is the end of our talk.

Acknowledgements

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